WAVE PROPAGATION IN ANISOTROPIC COMPOSITE PLATES

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ABSTRACT

This paper is concerned with the development of wave propagation based cost-effective ultrasonic, techniques for the nondestructive materials arid defects characterization in advanced structural composites. A theoretical model that captures the most significant features of wave phenomena in fiber reinforced composites is described. Laboratory tests are shown to yield results that arc in excellent agreement with those obtained from the theoretical model. An oblique insonification technique to determine the stiffness constants and thickness of graphite-epoxy composite laminates and their degradation under fatigue loading or thermal exposure is described. Another experiment using dual transducers in contact with composite laminates to detect and characterize acoustic emission sources is also described.

INTRODUCTION

Fiber-reinforced composites have highly desirable engineering properties including their relatively low weight, high stiffness and damage tolerance, that make them very attractive for aerospace and other modern structural applications. However, composites are very sensitive to the details of their manufacturing process and service conditions, both of which can introduce hidden defects in the structure resulting in a serious degradation in their performance. '1 'he safety and integrity of composite structures require careful monitoring, of their degradation throughout the life of the structure by nondestructive means. Development of effective practical nondestructive evaluation (NDE) methods for composites has posed a challenge to the research community for many years, due to the complex nature of the material, including their strongly anisotropic, heterogeneous and dissipative properties. Intensive recent research has resulted in considerable success in several fronts. In particular, the use of obliquely incident ultrasonic waves has been shown to provide quantitative information on defects, adhesion, and the elastic properties in laboratory specimens of a variety of composite materials. A practical technique based on dual transducers in contact with the specimen surface has also been developed for possible field applications. Both methods usc plate guided waves to interrogate the parts in contrast to bulk waves used in conventional ultrasonics. Guided waves arc highly sensitive to internal flaws as wc]] as to changes in the material properties (e.g., stiffness) of the specimen. 'l'his paper summarizes the principles behind these methods and the progress that has been made by the authors in applying them to characterize the material properties and degradation of graphite-epoxy composite laminates.

THE THEORETICAL MODEL

In order to develop reliable NDE techniques based on laboratory and field tests it is necessary to understand the detailed features of the waves that are associated with these tests through theoretical modeling. The behavior of ultrasonic waves propagating through fiber-composite materials is strongly affected by their inherent anisotropy, inhomogeneity and dissipative nature. Since the microstructure of these materials is extremely complex, it is necessary to introduce simplifications in constructing tractable theoretical models. One of these simplifications is derived from the fact that in structural composites the fiber diameter is small compared to the wavelengths associated with most ultrasonic tests. As an example, in graphite-epoxy, the fiber diameter is in the range of 5-10 pm and for frequencies up to 20 MHz the wavelength is larger than 100 pm. Thus the material can be treated as homogeneous and transversely isotropic, with symmetry axis along the fibers. Wave attenuation in polymer-matrix composites is caused by energy absorption in the viscoelastic matrix and scattering from the fibers as well as other inhomogeneities arising from the manufacturing process, Both of these effects can be modeled in the *frequency domain* by assuming that the overall stiffness constants of the material are complex and frequency-dependent. Thus the constitutive equation for a transversely isotropic elastic solid with its symmetry axis along the x1-axis (Figure 1) can be expressed in the form [1]

$$
\begin{vmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{,}^* & 0 & 0 & 0 \\ C_{,2} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{,2} & C_{23} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{55} \end{vmatrix} \begin{vmatrix} u_{1,1} \\ u_{2,2} \\ u_{3,3} \\ u_{2,3} + u_{3,2} \\ u_{1,3} + u_{3,1} \\ u_{1,2} + u_{2,1} \end{vmatrix}
\tag{1}
$$

where, $\sigma_{ij} exp(-i\omega t)$ *is* the Cauchy's stress tensor, $u_i, exp(-i\omega t)$ *is* the displacement vector, ω is the circular frequency and $u_{i,j} = (\partial u_i / \partial x_j)$. The constants $C_{11}, C_{12}, C_{22}, C_{23}, C_{55}$ are the *overall* stiffness constants of the composite and $C_{44} = (C_{22} - C_{23})/2$. We assume that the stiffness constants C_{ij} are complex and are related to the real stiffness constants c_{ij} of the material through

$$
C_{11} = c_{11}/(1 + ip\sqrt{(c_{55}/c_{11})}), \quad C_{22} = c_{22}/(1 + ip\sqrt{c_{55}/c_{22}})
$$

$$
C_{12} + C_{55} = (c_{12} + c_{55})/(1 + ip\sqrt{c_{55}/(c_{12} c_{55})})
\tag{2}
$$

$$
C_{44} = c_{44}/(1 + ip\sqrt{c_{55}/c_{44}}), \quad C_{55} = c_{55}/(1 + ip)
$$

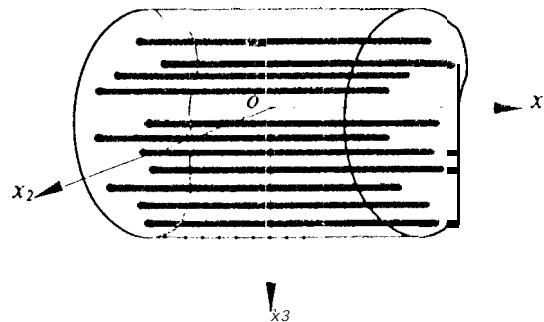where p is a frequency-dependent damping parameter [2].

Fig. 1. The transversely isotropic model of the fiber reinforced composite.

It should be noted that the material model described above is a generalization of a similar model of dissipative isotropic materials used in seismology [3]. For many isotropic solids, p is independent of frequency in a broad frequency range. In the case of fiber-reinforced composites the effect of wave scattering by the fibers is likely to be significant at higher frequencies. We include the effect of scattering by assuming that above a certain frequency ω_0, p becomes frequency dependent and that it can be expressed in the form

$$P \cdot p_0 + p_2 (\frac{J}{f_\wedge} - 1)^2 H(f - f_0) \tag{3}$$

where $f = \omega/2\pi$ is *the* frequency in cycles, $f_0 = \omega_0/2\pi$, $H(f)$ is the Heaviside step function, and PO, p_2, f are constants which determine the degree of decay in the amplitude of the waves with propagation distance. The first term in the right hand side of (3) represents dissipation due to internal friction and other thermodynamic effects, while the second term represents the attenuation due to wave scattering by the fibers and other inhomogeneities in the material. Moreover, wave attenuation due to scattering becomes more and more prominent as $f - f_0$ increases.

The material model described above has been used to solve a variety of elastodynamic problems involving unidirectional as well as multilayered and multiorientation composite laminates [4-6]. We omit the details of the solution procedure and present results for several problems of interest in NDE.

THE OBLIQUE INSONIFICATION EXPERIMENT

The basic idea behind this method is to transmit either a tone-burst or a short-pulse beam of acoustic waves onto the specimen immersed in water and to record and analyze the reflected signals (Figure 2). Using tone-burst signal, the phase velocity of the so called *leaky guided waves* in a water loaded composite laminate can be determined accurately as a function of frequency and this data can be analyzed to determine the resin dominated elastic constants and also to detect a variety of hidden defects. The use of the short pulse beam in an unidirectional composite plate allows identification of the various reflected signals from the plate surfaces, that can be analyzed to determine the stiffness and damping parameters of the material of the plate very accurately [4].
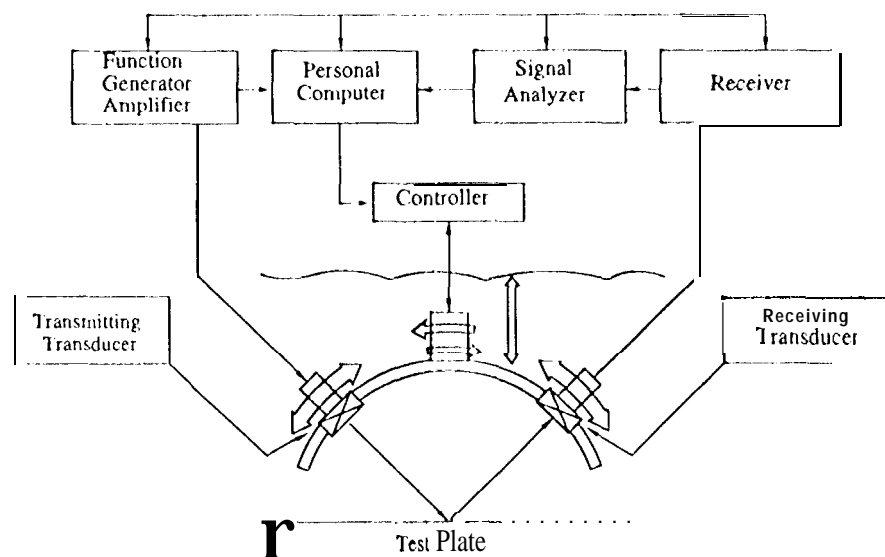

Fig. 2. The oblique insonification experiment.

Assume that a beam of acoustic waves is launched onto a composite plate of thickness H immersed in water (Figure 3). The angle of incidence is θ and the plane containing the transducer axes makes an angle ϕ with the plane containing the fibers in the top lamina and the plate normal. The angle θ can be changed in increments of about $0.05°$ by means of the controller in the range $10°<\theta<60°$, and ϕ can be changed in increments of $2°$ in the range $0°< \phi <360°$. The reflection coefficient from a unidirectional graphite-epoxy plate for an incident angle of $30°$ is shown in the right panel of Figure 3. The phase velocity of the leaky guided wave in the water loaded plate is calculated from Snell's law as $v = \alpha_0 /(sin\theta)$, where α_0 is the velocity of sound in water. For a fixed angle of incidence, leaky guided waves in the plate with phase velocity v are induced at frequencies at which the the reflection coefficient has its nulls or minima. By changing the incident angle and repeating the process, the *dispersion curves* of the leaky guided waves in the plate can be constructed, as shown in the bottom of the right panel in Figure 3. The locations of the minima in the reflection coefficients are highly sensitive to the thickness and the stiffness constants of the plate and are insensitive to the damping parameters as well as the presence of water in a broad frequency range. Thus the dispersion data can, in principle, be used to determine accurately these proper-tics and any changes in their values during service.

The phase velocity of guided waves in a composite laminate in absence of water loading is obtained from the theoretical model as a transcendental equation of the form [5],

$$G(V, \; f, c_{ij}, H) = 0 \qquad (4)$$

For a given dataset $\{f_k, v_k\}$, c_{ij} and H can be determined by minimizing the objective function

$$F(c_{ij}, H) = \Sigma w_k \, |G_k|^2 \qquad (5)$$

where w_k is a suitable weight function and G_k is the value of the dispersion function G at the kth data set. The minimization can be carried out through a variety of available optimization schemes; we have used the SIMPLEX algorithm to accomplish this.
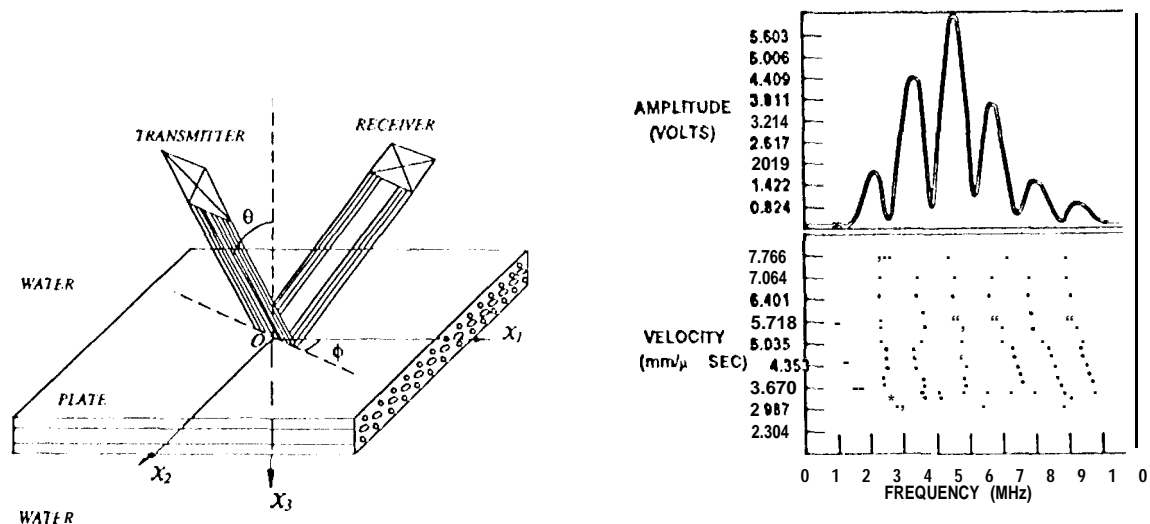
Computer display of reflected spectra and dispersion curve during sweep
● d data acquisition of LLW experiment

Fig. 3. Measurement of the dispersion curves of leaky guided waves in a composite laminate.

The results of inversion of dispersion data for an unidirectional and a cross-p] y graphite-epoxy laminate, each of 1 mm nominal thickness, are shown in Figure 4. Matched broadband transducer pairs with center frequency 5 MHz were used in the experiments. For the $[0]_8$ laminate, the data for propagation at 0° to the fibers, in the velocity range 3 - 6.5 mm/µs, were inverted through minimization of(5); *in this range the dispersion curves are not affected by the presence of water loading on the plate.* The inversion resulted in a plate thickness of 1.05 rmn and the stiffness constants given in Table 1. These values were then used to theoretically calculate the dispersion curves for the plate *without water loading* for propagation at 0", 45° and 90° to the fibers, for comparison with the measured dispersion curves of the leaky guided waves in the water loaded plate in the entire frequency and velocity ranges. It can be seen that the agreement between the measured and calculated curves is excellent except in a small range of velocities and frequencies, where the effect of water loading is significant. The " dispersion curves for a $[0, 90]_{2s}$ cross-ply laminate are shown in the right column of Figure 4; the material properties given in Table 1 were used in the calculations, It can be seen that although there is considerable amount of scatter in the data, the agreement between the calculated and measured dispersion curves is quite good. inclusion of water loading in the calculations improves the agreement but increases the computational effort [5].
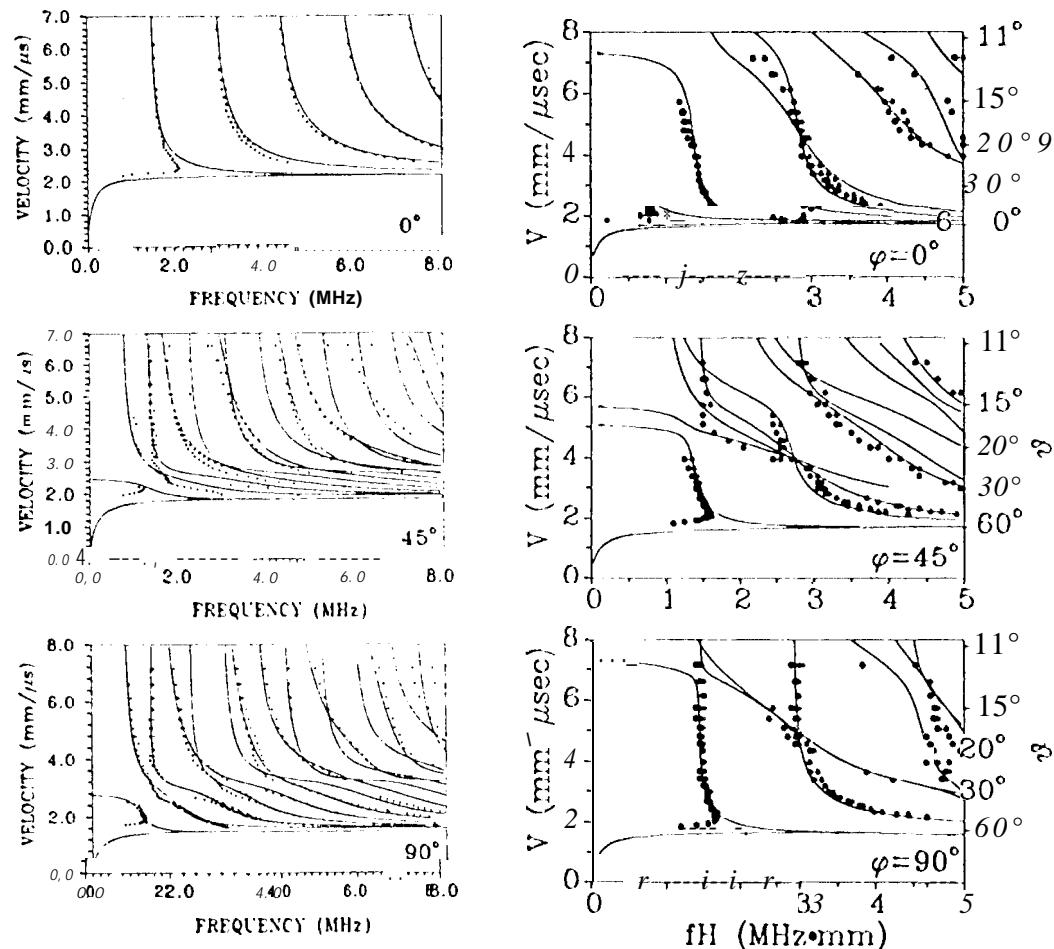
Fig. 4. Comparison between measured (·or ☉) and calculated (solid lines) dispersion curves for guided waves in a $[0]_8$ (left column) and a $[0, 90]_{2s}$ graphite-epoxy laminate (right column).

"TAll] JE 1: The inverted stiffness constants for the $[0]_8$ laminate.

ρ(g/cm²)	c_{11} (GPa)	c_{12} (GPa)	c_{22} (GPa)	c_{23} (GPa)	c_{55} (GPa)
1.578	166.07	7.09	16.19	7.52	7.79

To demonstrate the capability of the method to characterize materials degradation of composites, a sample made of AS4/3501 -6, [0]24 laminate was tested after it was subjected to heat treatment. The sample was exposed to a heat ramp from room temperature to $480°F$ for minutes, held at 480° F for 15 minutes, and then was taken out of the oven to cool in open air at room temperature. The sample was tested at a specific location before and after heat treatment. The measured dispersion curves are shown in Figure 5. It can be seen that there are distinct differences in the dispersion data for the specimen before and after heat treatment. Since the heat damage occurs mostly in the matrix, the effect is expected to be more pronounced in the matrix dominated. The constants $c_{11}, c_{12}, c_{22}, c_{23}$ and c_{55} obtained from the inversion process are 127.9, 6.32, 11,85, 6,92 and 7,43 GPa, before heat treatment, and 128.3, 6.35, 10.55, 6.9, and 7.71 GPa, after heat treatment. The most noticeable and significant change is in the stiffness constant c_{22}, which is the property most sensitive to variations in the matrix resulting in a reduction in the transverse Young's modulus.
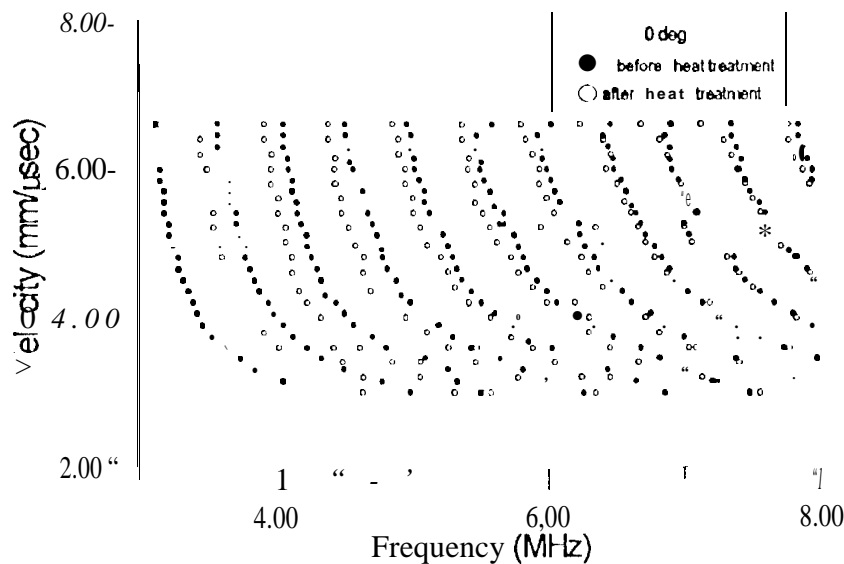

Fig. 5. The measured dispersion curves of a $[0]_{24}$ graphite-epoxy panel before and after heat treatment.

It should be noted that equation (4) is strongly nonlinear in c_{ij} and H, and its solution is nonunique. Thus extreme care must be taken in interpreting the numerical results obtained from the inversion of the dispersion data. On the basis of extensive parametric studies of equation (4) wc have concluded that only the thickness and the matrix dominated constants, c_{22}, c_{23} and c_{55} can bc determined accurate] y from the inversion of the dispersion data,"1 'his is due to the fact that the dispersion function G is not very sensitive to the fiber dominated constants, c_{11} and c_{12}. These two constants can bc determined accurately from the travel times and amplitudes of the reflected short-pulse signals in the oblique insonification experiment. "l'he details of this procedure can be found in [4] and are omitted.

In order to determine the damping constants of the material, it is necessary to calculate the amplitudes of the reflected waves. However, in the experiment the precise nature of the incident field is unknown. Moreover, the reflected field is influenced by a variety of factors including the transducer response, geometrical spreading and diffraction. These factors are, in general, difficult if not impossible to model accurately. in order to incorporate these unknown factors in the theoretical calculations the plane wave response spectrum is multiplied by the sign reversed spectrum of the reference pulse, constructed from the first pulse present in the reflected signal. Finally, the resulting spectrum is inverted by FFT to give the time history of the reflected field and multiplied by a constant scaling factor

determined by matching the first (top surface) reflected pulse to that in the measured signal, It should be noted that the reference signal needs to be determined only once for each specimen. The result for a 25 mm thick unidirectional graphite/epoxy plate is presented in Figure 6. The stiffness constants given in Table 1 were used in the calculations. The source function used in the calculations is given at the top and the calculated reflected signals assuming perfect elasticity are compared with measured data in the left column of the figure. It can be seen that the calculated reflected pulses from the interior of the plate are much larger than the corresponding measurements. The calculations for the dissipative case using the values of the constants, $f_0 = .3$, $p_0 = .005$ and $p_2 =- .0015$, were found to yield the best visual fit in these pulses; the results are shown in the right column. These results indicate that the dissipative properties of the composite need to be incorporated in the theoretical model for accurate prediction of the amplitudes of the pulses reflected from the interior of the plate. The prediction based on the model of wave attenuation proposed here are in excellent agreement with experimental data for composite specimens of different thicknesses. The value of p_0 determined from this study is consistent with the values of the qualit y factor for most homogeneous and isotropic materials. Tests on samples of different thicknesses showed that the value of p_2 increases with increasing laminate thickness, indicating the presence of inhomogeneities introduced during processing of thicker laminates.
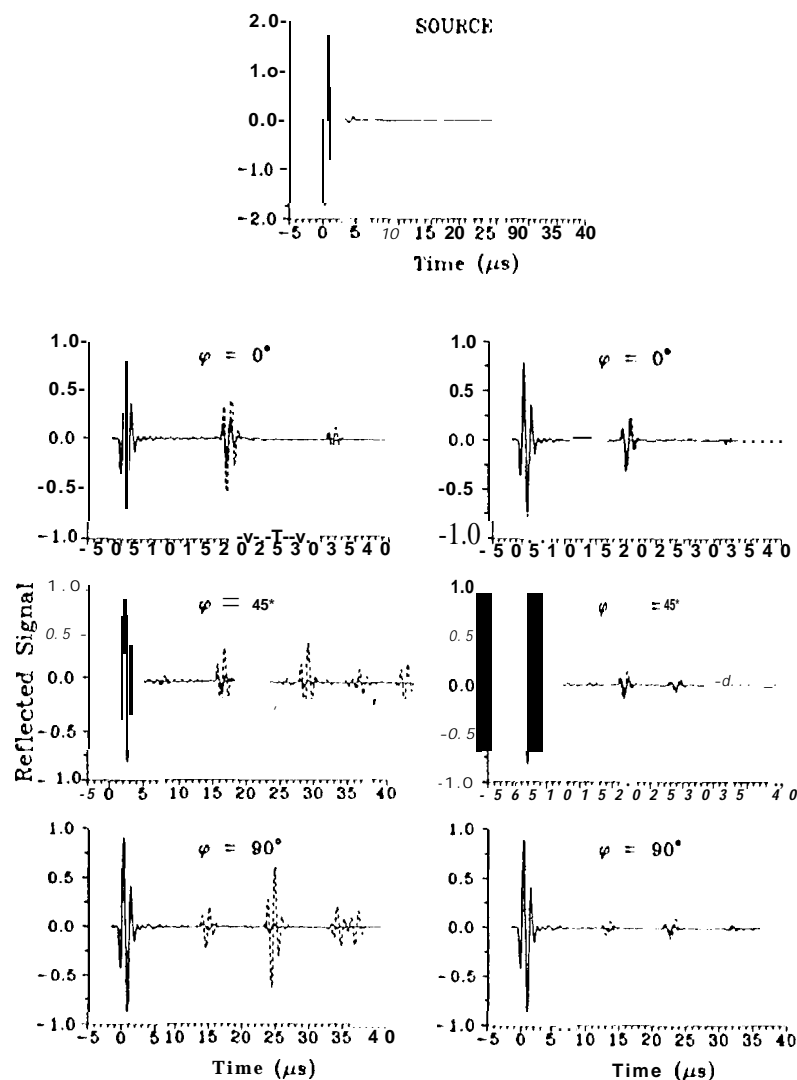

Fig. 6. Measured (solid curves) and calculated (dashed curves) reflected signals from a unidirectional 25 mm thick graphite-epoxy plate in water for incident angle O = 20'. Calculated results in the left column assume no dissipation while those in the right column include dissipation,

CONTACT EXPERIMENT

The laboratory set up for the contact experiment is shown in Figure 7. The source is a surface load, e.g., a pencil break or a pulse from a function generator and amplified by a gated pulse amplifier, A number of identical broadband transducers arc used as receivers, and a *Fracture Wave Detector*, made by Digital Wave Corporation is used for data acquisition. This technique can be used to characterize materials degradation as well as acoustic emission sources in composite parts in service conditions,

The theoretical simulation of the contact experiment requites calculation of the response of the laminate to concentrated or distributed surface loads involving three dimensional models. The solution to a number of such problems has been given by Lih and Mal [6]. **Figures** 8 shows comparison between measured and calculated surface displacements on a unidirectional composite plate due to a pencil break source. The agreement can be seen to be very good.

Finally the theoretical model was used to calculate the displacements produced by acoustic emission sources within composite laminates. Typical such sources consist of matrix cracking, fiber breakage and delamination. Each of these source types were represented by a suitable moment tensor, and the motion produced by couple system was calculated by means of the three dimensional theoretical model described earlier. A typical result of these calculations for a unidirectional composite plate of 1 mm thickness is shown in Fig. 9. The calculated waveforms are qualitatively similar to those observed in laboratory experiments. However, the model is too idealistic to yield quantitative results. Further research to improve the model as well as additional experiments are currently in progress.
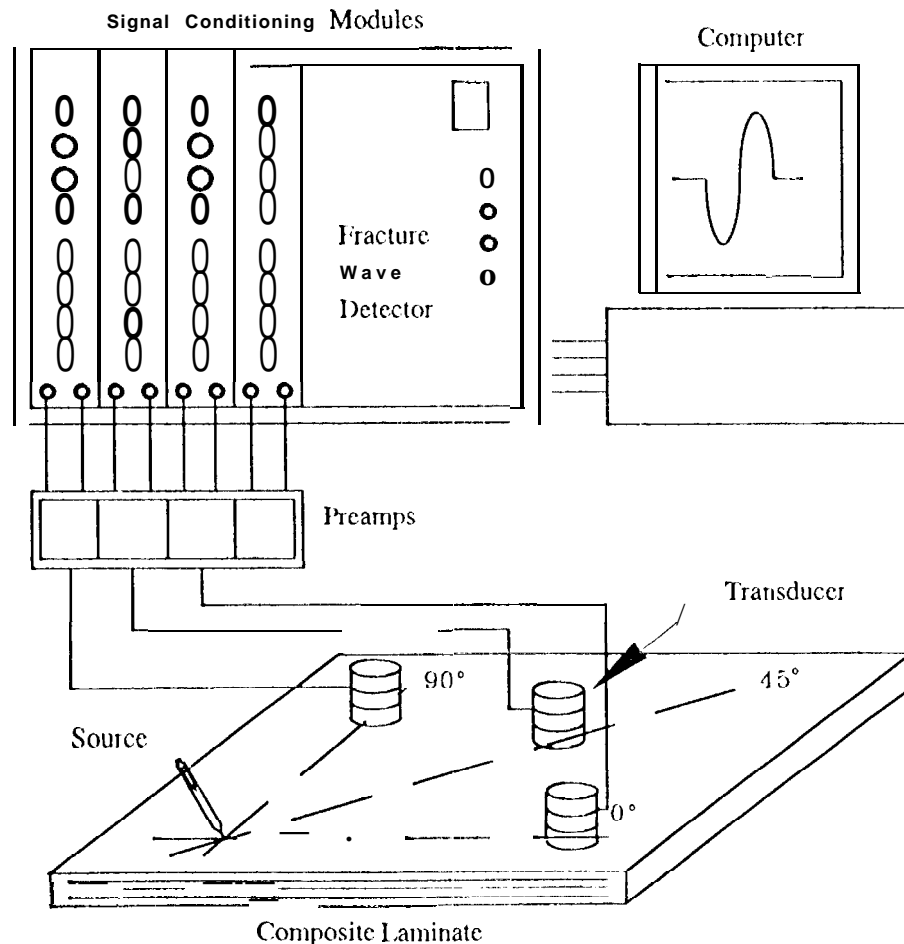

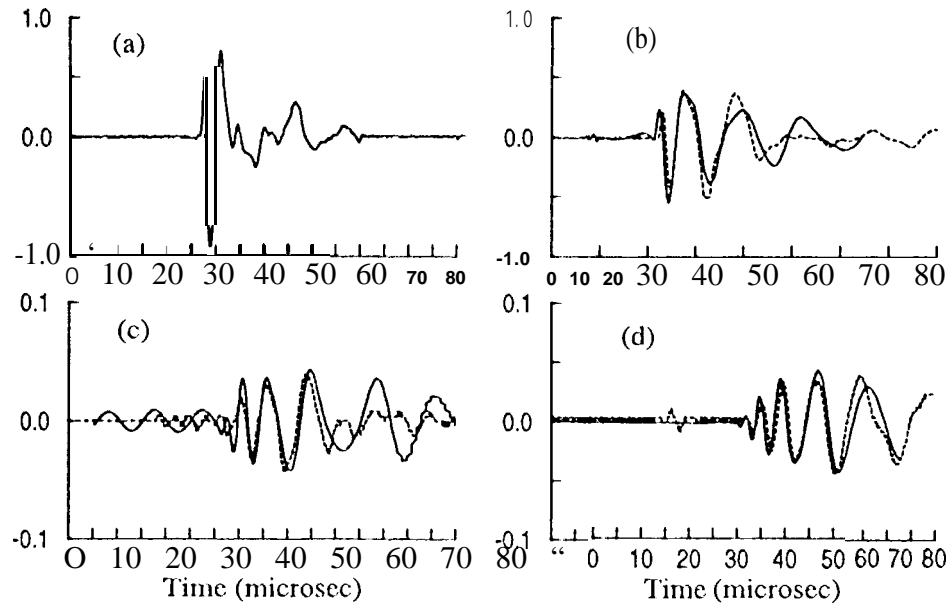Fig. 7. Schematic diagram of the contact experiment.

Fig. 8. Comparison between measured and calculated surface displacements in a unidirectional graphite-epoxy plate of thickness 1 mm due to a surface pencil break source.
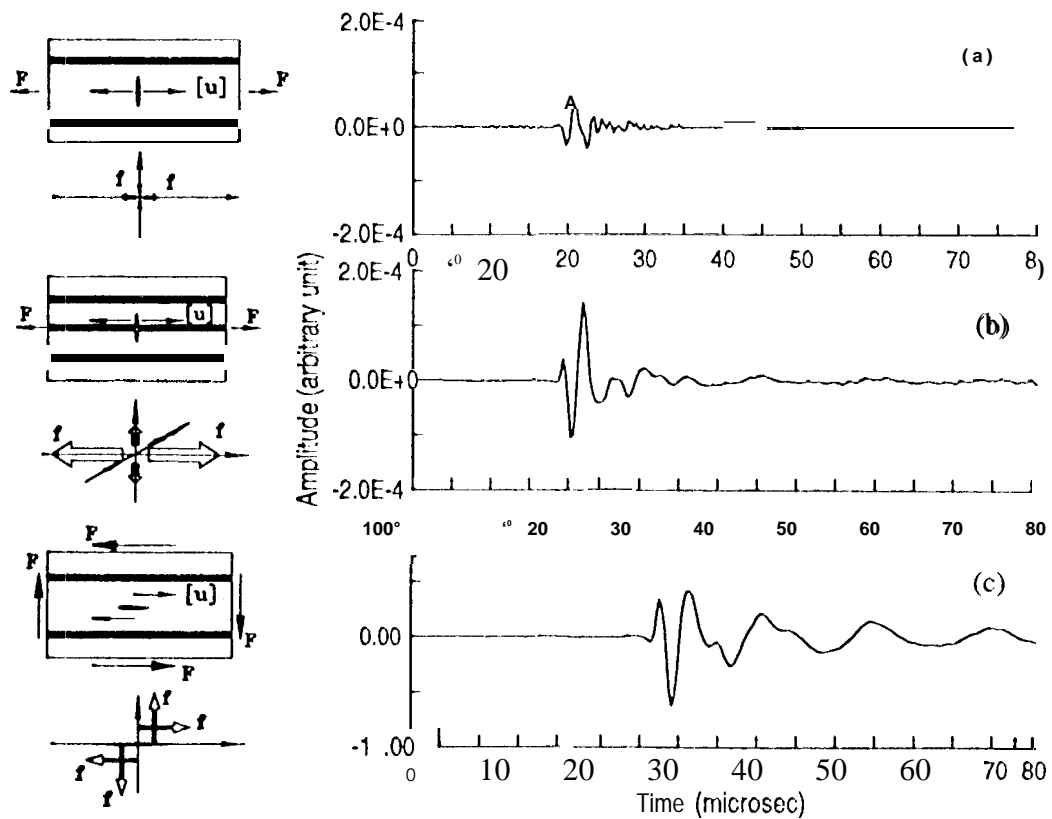

Fig. 9. Calculated surface displacements on a unidirectional composite plate produced by matrix cracking, delamination and fiber break within the plate.

CONCLUDING REMARKS

An overview of the theoretical and experimental research carried out by the author's group at UCLA is provided in this paper. No attempt is made to give an exhaustive review of similar work conducted by other groups. The main objective of our research is to develop the knowledge base required for the characterization of the degradation of structural composites when they are subjected to service conditions. It can be seen that although considerable progress has been made in developing potentially useful NDE techniques in recent years, more research is needed to refine them and to make thm applicable in field environments,

ACKNOWLEDGMENTS

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